Fiscal Policy Rules, Budget Deficits and Forecasting Biases*

Guido Baldi†‡

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Abstract

In this paper, we analyze the impact fiscal policy rules have on budget deficits and forecasting biases in official budget outlooks. Persistent budget deficits and over-optimistic budget forecasts have been observed in many countries in the past, especially in the euro area. To prevent such developments from happening in the future, fiscal rules have been revised or implemented with the aim to strengthen both preventive (ex-ante) and corrective (ex-post) elements of fiscal rules frameworks. Do such ex-ante and ex-post rules differ in their effects? In an attempt to answer this question, we build a two-period model and distinguish between ex-ante rules that apply to budget forecasts and ex-post rules that apply to realized budget deficits. Our model indicates that effectively enforced ex-post rules are more effective than ex-ante rules at reducing budget deficits. Interestingly, ex-ante rules differ from ex-post rules in their effects on forecasting biases. Only ex-post sanctions reduce forecasting biases, while ex-ante rules have no impact on such biases. In addition, we show that political stability and the size of government increase the effectiveness of fiscal rules. If, however, financial markets have a disciplining effect on governments, the effectiveness of fiscal rules is reduced. Our results imply that if fiscal policy rules cannot be effectively enforced, reforming other areas such as electoral rules or financial market regulations might be a more promising approach to ensuring sound public finances than fiscal policy rules.

JEL CLASSIFICATION: E6, H6, H11
KEYWORDS: Fiscal Policy Making, Fiscal Policy Rules, Forecasting Bias

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†University of Bern, Department of Economics, Address: Schanzeneckstr. 1, CH-3000 Bern, Switzerland. Email: guido.a.baldi@gmail.com

‡German Institute for Economic Research (DIW Berlin), Research Cluster Macroeconomics and Financial Markets, Berlin, Germany. Address: DIW, Mohrenstr. 58, D-10117, Berlin, Germany

1 Introduction

What impact do fiscal policy rules have on the size of budget deficits and forecasting biases in official budget outlooks? In this paper, we present a simple two-period model to investigate these issues. The model that incorporates budget forecasts and allows for the possibility of biased forecasts. Our main goal is to analyze whether ex-ante rules, which apply to budget forecasts, have different effects than ex-post rules, which apply to the realized budget deficit. The model is inspired by earlier theoretical work conducted by Milesi-Ferretti (2004) and contributes to the discussion about the so-called "Fiscal Compact" and the revised Stability and Growth Pact (SGP) in the European Union.[1] The provisions stipulated in this fiscal policy rules framework are supposed to strengthen and overcome the limitations of the previously existing rules in the euro area and facilitate the enforcement of the fiscal rules both ex-ante and ex-post. In the years before the crisis, many euro member countries tended to postpone fiscal adjustments through optimistic forecasts that the deficits would soon fall below the deficit limits of the SGP. When actual deficits were actually higher than forecasted, bad luck or unforeseen circumstances served as excuses for the governments. This, along with other factors, allowed governments to escape sanctions ex-post. The revised fiscal rules framework is supposed to be effective because it obliges countries to take increased responsibility for fiscal management. Fiscal rules are intended to be more binding by adopting them at the national level, by strengthening both the preventive and corrective arms of the fiscal rules, and by making sanctions more likely both ex-ante and ex-post. However, several authors, e.g., Buti and Pench (2012), identified potential limitations of this framework, especially regarding the enforcement mechanisms.

There are several papers that investigate the effects of the fiscal criteria on public finances - especially when it comes to those rules stipulated in the Maastricht Treaty. For instance, Gali and Perotti (2003) estimate fiscal reaction functions and find that euro area membership did not cause fiscal policy to become less counter-cyclical than in those countries that did not intend to become a member of the euro area. Also the results reported by Ballabriga and Martinez-Mongay (2003) imply that the Stability and Growth Pact had little impact on fiscal policies. An important issue is whether fiscal rules can prevent governments from using over-optimistic budget forecasts to delay necessary fiscal adjustments and to circumvent the provisions stated in the fiscal rules. In general, budget forecasts may be biased on both the optimistic and pessimistic side. However,

[1] For a discussion of the "Fiscal Compact", see e.g., Bird and Mandilaras (2013).

the empirical literature mostly finds that governments exhibit an optimism bias in forecasting. Over-optimistic budget forecasts and the use of creative accounting techniques by governments to circumvent the constraints on deficits and debt are often argued to have significantly contributed to fiscal problems in the past, especially in the euro area (see e.g. Koen and van den Noord (2005), von Hagen and Wolff (2006), Buti, Martins and Turrini (2007) and Frankel and Schreger (2013)). The existence of forecasting biases has also been detected for other countries. For the US, Auerbach (1999) finds that the forecasts of the US Office of Management and Budget (OMB) tended to overestimate revenues during the period from 1986 to 1993. McNab, Rider and Wall (2007) find that forecasts one year ahead of the OMB were biased from 1963 to 2003. Presumably, as they argue, this bias may have been strategic to achieve particular goals, such as to increase spending or cut taxes. A pessimism bias may also occur, that is, a bias for overly pessimistic forecasts, which is documented to have prevailed in Switzerland (see, e.g., Chatagny and Soguel (2012)) and which could have been strategic and related to fiscal rules. For the US, Auerbach (1999) finds a tendency of the OMB in the US to underestimate revenues for the period 1993 to 1999.[2] Our model shows that effectively enforced fiscal rules can improve public finances. The size of budget deficits is reduced by both ex-ante and ex-post sanctions. However, only ex-post sanctions reduce forecasting biases.

In addition, we show how the effects of fiscal policy rules depend on other factors, an area that has been rarely discussed in the literature. Our paper stresses the importance of the institutional environment, an issue that has been emphasized by Hall and Gingerich (2009) and, in the context of fiscal policy rules, by Mause and Groeteke (2012). This implies that the effectiveness of rules depends on the institutional and political structures in those countries subjected to such rules. Consistent with this, the findings of our paper emphasize that the effectiveness of fiscal policy rules should not be viewed in isolation. For example, actors in the financial markets may react to the state of public finances and risk premia on sovereign debt may depend on the presence and strength of fiscal rules (see e.g., Iara and Wolff (2014)). A major reason for the adoption of fiscal rules in the European Union was the concern that market discipline alone would not have a sufficient disciplinary effect on public finances. In addition, we show that the effectiveness of rules depends on the degree of political stability. Since electoral rules and political systems are very different from country to country (including in the European Union), the frequency of government changes and the degree of political competition varies across countries.

[2]Other papers dealing with biased budget forecasts include Jonung and Larch (2004), Brueck and Stephan (2006) and Beetsma, Giuliodori and Wierts (2009). For a comprehensive review of the literature, see Frankel and Schreger (2013).

Our results suggest that political stability increases the effectiveness of fiscal rules, which raises implications that have so far been rarely discussed in the literature and among policy-makers. We also investigate whether the size of government impacts on the effectiveness of fiscal rules, which is a relevant especially for countries in the European Union where the size of government considerably varies across countries.

The remainder of this paper is organized as follows. In Section 2, we present a simple model to analyze the relations among fiscal rules, budget deficits and forecasting biases. Section 3 discusses the implications of our model, and Section 4 presents the conclusions.

2 The Model

We consider a two-period model with a government maximizing its utility function. The government derives utility from government spending in period 1 and 2 (S_1 and S_2). From these expenditures, the government realizes a marginal utility increase of a, which determines the size of government. We focus on spending as the choice variable of the government and assume that government spending is financed by constant lump-sum taxes T. Spending in excess of tax revenues is financed by government debt. In our model, budget deficits lead to sanctions that are derived from fiscal policy rules. To investigate potential reasons for a forecasting bias, we must distinguish between officially published budget forecasts and non-published actually intended budget figures. Following this reasoning, we denote the officially forecasted deficit by F_1, which may be a biased forecast; while D_1 is the actually intended deficit that the public does not observe. This is not necessarily the final realized budget deficit, as this may additionally be influenced by an exogenous shock ϵ_1. Only F_1, which is the official forecast and the final budget outcome $B_1 = D_1 + \epsilon_1$ are observed by the public. Under the assumption that the shock to the final budget is exogenous and has an expected value of zero, we drop ϵ_1 for the following discussion.

Because we are considering a two-period model, there is the probability p that a government remains in power after the election at the end of period 1 and is still in power in period 2. Thus, p is the time discount factor used by the government and can also be interpreted as capturing the degree of political stability. In period 2, the government pays back the public debt. Furthermore, because the government pretends to run a deficit of F_1 until the end of period 1, the additional deficit given by $D_1 - F_1$, financed at the very end of period 1 or the beginning of period 2, is discounted by the government by p.

An officially forecasted deficit published at the beginning of period 1 that is higher than the structural or expected deficit of the public (D^{Exp}) is assumed to lead to a quadratic welfare loss in period 1, whose size is determined by κ^a. One may, for example, consider financial markets reacting to a deterioration of public finances, which increases risk premia on government bonds and raises the general interest rate level and thus leads to crowding-out effects in the rest of the economy. The quadratic form chosen for the welfare loss has the convenient property that the first-order conditions will be linear. If the government forecasts surpluses (or if the deficit is lower than the generally expected deficit), κ^a switches its sign and a welfare gain appears. In period 2, financial market reactions lead to a welfare loss determined by κ^p for deviations of the actual deficit from the officially forecasted deficit in the event the actual deficit is higher than the forecasted deficit. Analogous to κ^a, κ^p switches its sign if the actual deficit appears to be lower than the forecasted deficit. An extreme example of a deviation between actual and previously forecasted deficits that lead to considerable market reactions occurred in Greece in 2009, when the government revealed news about higher than previously communicated public deficits and debt. Taken together, the utility function of the govenment in this two-period model is given by:

$$\begin{aligned} U_1 + pU_2 =& aS_1 - \frac{\kappa^a}{2}\left(F_1 - D^{Exp}\right)^2 \\ &+ p\left(aS_2 - \frac{\kappa^p}{2}(D_1 - F_1)^2\right) \end{aligned} \tag{1}$$

κ^a and κ^p are specified as:

$$\begin{aligned} \kappa^a > 0 \quad &\text{if} \quad \left(F_1 - D^{Exp}\right) > 0 \\ < 0 \quad &\text{if} \quad \left(F_1 - D^{Exp}\right) < 0 \\ \kappa^p > 0 \quad &\text{if} \quad (D_1 - F_1) > 0 \\ < 0 \quad &\text{if} \quad (D_1 - F_1) < 0 \end{aligned}$$

In our model, forecasted and realized budget deficits lead to financial sanctions that are derived from fiscal policy rules. Sanctions that apply to officially forecasted deficits are called *ex-ante* sanctions. *Ex-post* sanctions are those sanctions that apply to actually realized budget deficits. Both these financial sanctions are proportional to the size of the deficit. The ex-ante sanctions on officially forecasted deficits are determined by η^a, and

η^p denotes ex-post sanctions on realized deficits. In general, one should also consider the probability that these financial sanctions are actually imposed. This is captured by assuming that η^a and η^p measure the total strength of a financial sanctions, which is the product of the size of the sanctions and the probability of enforcement. These measures of the strength of financial sanctions are incorporated in the budget constraint of the government. For ease of discussion, we consider a symmetric sanction and benefit scheme, where deficits are sanctioned and budget surpluses are rewarded. Assuming that productivity-enhancing government outlays spent in period 1 become productive in period 2, the government budget constraints for periods 1 and 2 are written as:

$$D_1 + T = S_1 + \eta^a \left(F_1 - D^{Exp}\right) \tag{2}$$

$$-D_1 + T = S_2 + \eta^p (D_1 + \epsilon_1) \tag{3}$$

The government maximizes its utility function subject to the budget constraints in (2) and (3). This yields the following first-order conditions for S_1, S_2, F_1 and D_1, where λ_1 and λ_2 denote the Lagrangian multiplier in periods 1 and 2:

$$0 = a - \lambda_1 \tag{4}$$

$$0 = a - \lambda_2 \tag{5}$$

$$0 = -\kappa^a \left(F_1 - D^{Exp}\right) - \lambda_1 \eta^a + p\kappa^p \left(D_1 - F_1\right) \tag{6}$$

$$0 = -p\kappa^p (D_1 - F_1) + \lambda_1 - p\lambda_2 (1 + \eta^p) \tag{7}$$

3 Analysis of the Model

In this section, we analyze the impact of fiscal rules and institutional characteristics on budget deficits and forecasting biases. For ease of discussion and without loss of generality, we focus on cases where deficits are forecasted. This implies that discussing lower deficits is analogous to discussing higher surpluses. From the first-order condition for the officially announced forecasts D_1 in (7), we obtain an expression for the forecasting bias $D_1 - F_1$. If there is an optimism bias, we obtain:[3]

[3] If there is a pessimism bias, we obtain:

$$D_1 - F_1 = \frac{-a\left(1 - p(1 + \eta^p)\right)}{p\kappa^p}$$

$$D_1 - F_1 = \frac{a\,(1 - p(1 + \eta^p))}{p\kappa^p} \tag{8}$$

From (8), it is evident that there is a systematic difference between unofficially intended and officially forecasted budget deficits. In a next step, we derive expressions for the officially forecasted deficit F_1 and the actually intended deficit D_1. Combining (6) and (8) we obtain, if a budget deficit is forecasted:

$$F_1 = \frac{a\,(1 - p\,(1 + \eta^p) - \eta^a)}{\kappa^a} + D^{Exp} \tag{9}$$

Because we assumed that $1 - p(1 + \eta^p) > 0$, there is a deficit forecasted if $D^{Exp} + \frac{a(1-p(1+\eta^p)-\eta^a)}{\kappa^a} > 0$. By combining (8) and (9), one can derive an expression for D_1:

$$D_1 = \frac{a\,(1 - p(1 + \eta^p))}{p\kappa^p} + \frac{a\,(1 - p(1 + \eta^p) - \eta^a)}{\kappa^a} + D^{Exp} \tag{10}$$

The expressions in (8), (9) and (10) allow us to analyze how fiscal rules and other institutional factors impact budget deficits and forecasting biases.

Proposition 1: Financial ex-ante sanctions for forecasted budget deficits lower the officially forecasted deficit as well as the actually intended budget deficit by the same amount. Thus, they do not impact the forecasting bias.

Proof: See Appendix

Proposition 2: Ex-post sanctions for realized budget deficits reduce actually intended deficits and forecasted deficits. Because actual deficits decrease more than forecasted deficits, the optimism bias in the forecasts decreases.

Proof: See Appendix

For a stable solution, we assume $1 - p(1 + \eta^p) > 0$.

Propositions 1 and 2 state that both ex-ante and ex-post rules decrease budget deficits. However, the size of the deficit reduction is higher for ex-post rules than for ex-ante rules.[4] In addition, ex-post rules decrease forecasting biases, while ex-ante rules do not impact these biases. As ex-ante rules only apply to forecasted deficits, they decrease the level of forecasted and realized deficits by the same amount and do not affect the decision to bias forecasts. Ex-post rules, however, lower the forecasting bias in addition to lowering the budget deficit. Interestingly, this reduction in the forecasting bias is achieved by lowering budget forecasts more than the actually intended budget deficit. Thus, the results of our model show that ex-post rules are better suited than ex-ante rules to reduce budget deficits and forecasting biases. This implies that policy-makers in the European Union and elsewhere should primarily focus on enforcing the "corrective" elements of their fiscal rules framework rather than the "preventive" elements.

Proposition 3: If the government experiences a higher marginal utility of spending (i.e. when the size of government increases), the effects of both ex-ante and ex-post rules are reinforced. That is, both forecasted and actual deficits are further decreased. In the case of ex-post rules, the optimism bias further decreases, because deficits are decreased more than forecasts.

Proof:

See Appendix

Proposition 3 shows that if the size of government increases, effectively enforced fiscal rules have a stronger impact on the reduction of deficits and an interesting difference emerges between ex-ante and ex-post rules. When the size of government increases, forecasted deficits are lowered more by ex-ante rules. In contrast, actual deficits and forecasting biases are decreased by a larger amount under ex-post rules. As ex-ante rules do not impact the forecasting bias, altering the size of government does not change the effects of ex-ante rules on this bias. These results imply that if large governments adopt fiscal policy rules, they can more effectively ensure sound public finances.

[4] If $\kappa^a/\kappa^p > 1-p$, which always holds under the assumption that $\kappa^a > \kappa^p$.

Proposition 4: An increase in political stability does not impact the effectiveness of ex-ante rules on forecasted and actual deficits. However, higher political stability increases the effectiveness of ex-post rules regarding the forecasted and actual deficit, but not the forecasting biases.

Proof:

See Appendix

Political stability increases the deficit-reducing effects of ex-post fiscal policy rules, but does not impact on the effectiveness of ex-ante rules. Thus, our findings imply that fiscal policy rules may not be suitable to achieve sound public finances in politically unstable countries. For example, frequent elections or cabinet reshuffles reduce the effectiveness of fiscal rules. In such countries, other institutional mechanisms as, for example, the competencies of the finance and spending ministers may be more important to ensure sound public finances.

Proposition 5: Greater ex-ante financial market pressures reduce the effectiveness of ex-ante and ex-post sanctions on forecasted and actual deficits. Ex-ante financial market pressures do not impact forecasting biases.

Proof: See Appendix

Proposition 6: Greater ex-post financial market pressures reduce the effectiveness of ex-post sanctions on the deficit and forecasting biases and have no effect on budget forecasts or on the effectiveness of ex-ante sanctions.

Proof: See Appendix

Propositions 5 and 6 state that greater financial market pressures reduce the marginal effects of fiscal policy rules. This implies that financial markets that react to a deterioration of public finances by demanding higher risk premia do not only reduce the need to adopt fiscal rules, but they also decrease the effectiveness of such rules. Hence, fiscal

policy rules should be adopted in cases where financial markets do not effectively constrain fiscal policy by demanding higher risk premia. At some level, fiscal policy rules and financial market pressures are substitutes for one another.

4 Conclusions

In this paper, we analyzed the impact of fiscal policy rules on budget deficits and forecasting biases in official budget outlooks. Our model provides insights on whether fiscal rules have an effect on budget deficits and biases in budget forecasts. We also take into account that other factors impact on the effectiveness of fiscal policy rules. Our findings imply that while ex-post rules are more effective in lowering budget deficits than ex-ante rules, only ex-post rules lower the optimism bias in forecasting. In addition, institutional factors may have important and non-trivial effects on the size of budget deficits and potential optimism biases in forecasts. The results of the model imply that fiscal policy rules are most effective when the size of government is large and the political environment is stable. Based on our findings, it is suggested that future research seeks to endogenize and further investigate the relation between public finances, fiscal policy rules and the institutional environment.

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Appendix: Proofs of Propositions Section 4

Proposition 1

Proof.

$$\frac{\partial F_1}{\partial \eta^a} = \frac{-a}{\kappa^a} < 0$$
$$\frac{\partial D_1}{\partial \eta^a} = \frac{-a}{\kappa^a} < 0$$
$$\frac{\partial \left(D_1 - F_1\right)}{\partial \eta^a} = 0$$

□

Proposition 2

Proof.

$$\frac{\partial F_1}{\partial \eta^p} = -\frac{ap}{\kappa^a} < 0$$
$$\frac{\partial D_1}{\partial \eta^p} = \frac{a}{\kappa^a \kappa^p}\left(-\kappa^a - p\kappa^p\right) < 0$$
$$\frac{\partial (D_1 - F_1)}{\partial \eta^p} = \frac{-a}{\kappa^p} < 0$$

□

Proposition 3

Proof.

$$\frac{\partial^2 F_1}{\partial \eta^a \partial a} = \frac{-1}{\kappa^a} < 0$$
$$\frac{\partial^2 D_1}{\partial \eta^a \partial a} = \frac{-1}{\kappa^a} < 0$$
$$\frac{\partial^2 \left(D_1 - F_1\right)}{\partial \eta^a \partial a} = 0$$

$$\frac{\partial^2 F_1}{\partial \eta^p \partial a} = -\frac{p}{\kappa^a} < 0$$
$$\frac{\partial^2 D_1}{\partial \eta^p \partial a} = \frac{-1}{\kappa^p} - \frac{p}{\kappa^a} < 0$$
$$\frac{\partial^2 (D_1 - F_1)}{\partial \eta^p \partial a} = \frac{-1}{\kappa^p} < 0$$

□

Proposition 4

Proof.

$$\begin{aligned}
\frac{\partial^2 F_1}{\partial \eta^p \partial p} &= -\frac{a}{\kappa^a} < 0 \\
\frac{\partial^2 D_1}{\partial \eta^p \partial p} &= -\frac{a}{\kappa^a} < 0 \\
\frac{\partial^2 \left(D_1 - F_1\right)}{\partial \eta^p \partial p} &= 0 \\
\frac{\partial^2 D_1}{\partial \eta^a \partial p} &= 0 \\
\frac{\partial^2 F_1}{\partial \eta^a \partial p} &= 0 \\
\frac{\partial^2 \left(D_1 - F_1\right)}{\partial \eta^a \partial p} &= 0
\end{aligned}$$

□

Proposition 5

Proof.

$$\begin{aligned}
\frac{\partial^2 F_1}{\partial \eta^a \partial \kappa^a} &= \frac{a}{\left(\kappa^a\right)^2} > 0 \\
\frac{\partial^2 D_1}{\partial \eta^a \partial \kappa^a} &= \frac{a}{\left(\kappa^a\right)^2} > 0 \\
\frac{\partial^2 \left(D_1 - F_1\right)}{\partial \eta^a \partial \kappa^a} &= 0 \\
\frac{\partial^2 F_1}{\partial \eta^p \partial \kappa^a} &= \frac{ap}{\left(\kappa^a\right)^2} > 0 \\
\frac{\partial^2 D_1}{\partial \eta^p \partial \kappa^a} &= \frac{ap}{\left(\kappa^a\right)^2} > 0 \\
\frac{\partial^2 \left(D_1 - F_1\right)}{\partial \eta^p \partial \kappa^a} &= 0
\end{aligned} \tag{11}$$

□

Proposition 6

Proof.

$$\frac{\partial^2 F_1}{\partial \eta^a \partial \kappa^p} = 0$$
$$\frac{\partial^2 D_1}{\partial \eta^a \partial \kappa^p} = 0$$
$$\frac{\partial^2 (D_1 - F_1)}{\partial \eta^a \partial \kappa^p} = 0$$
$$\frac{\partial^2 F_1}{\partial \eta^p \partial \kappa^p} = 0$$
$$\frac{\partial^2 D_1}{\partial \eta^p \partial \kappa^p} = \frac{a}{(\kappa^p)^2} > 0$$
$$\frac{\partial^2 (D_1 - F_1)}{\partial \eta^p \partial \kappa^p} = \frac{a}{(\kappa^p)^2} > 0$$

□